On the Dynamics of Two Dimensional Array Beam Scanning via Coupled Oscillators

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Agenda

- Introduction and Background
- The Continuum Model in Two Dimensions
- The Green's Function
- Solution for Beamsteering
- Example Beam Scanning Behavior
- Concluding Remarks

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This presentation will begin with a description of the previous published work contributing to the results reported here. The previously developed one dimensional continuum model will be generalized to two dimensions and a Green's function for the resulting differential equation will be obtained as an eigenfunction expansion. This will be used to obtain dynamic solutions relevant to the steering of the radiated beam. Finally, some remarks concerning limitations on the interoscillator phase difference will be provided.

Introduction

- Concept due to Liao and York [IEEE Trans. MTT-41, pp. 1810-1815,October 1993].
    - Linear array of VCOs.
    - Antisymmetric detuning of end oscillators.
    - Linear aperture phase with variable gradient.
    - Analysis via numerical solution of a system of first order nonlinear differential equations based on Adler's theory of injection locking.

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The fundamental concept of steering phased array beams by appropriately tuning the end oscillators of a linear array originated with Liao and York in 1993. They suggested that linear phase progressions along the array could be established if the end oscillators were antisymmetrically detuned from the ensemble frequency. They also verified this experimentally at X-band in an array in which the coupling was achieved through the electromagnetic coupling between the radiating elements. Since this was a function of the element spacing, the design was over constrained.

This analysis of the array took the form of numerical solution of a system of first order nonlinear differential equations derived using Adler's theory of injection locking. This made intuitive understanding of the dynamics difficult.

Introduction (Continued)

- **Continuum model by Pogorzelski and York [IEEE AP-S Symposium Digest, pp. 324-327, July 1997].**
    - Continuous phase function of continuous variable indexing oscillators.
    - Governed by second order partial differential equation.
    - Steady state is analogous to electrostatics.
        - Detuning=Charge
        - Phase = Potential

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Beginning around 1996, Pogorzelski and York developed a continuum model of coupled oscillator arrays in which the phase is described by a continuous function of a continuous variable which, when it takes on integer values, indexes the oscillators of the array. The behavior of this continuous function is governed by a second order linear partial differential equation which can be solved analytically using standard techniques. This greatly enhances insight into the dynamics of such arrays and the relationship between the behavior of the phase and tuning of the oscillators.

In applying this to the beamsteering problem, it was noted that an analogy with electrostatics is evident in which the phase plays the role of electrostatic potential and the tuning plays the role of electric charge density. Here again intuitive understanding is enhanced.

The M by N Array
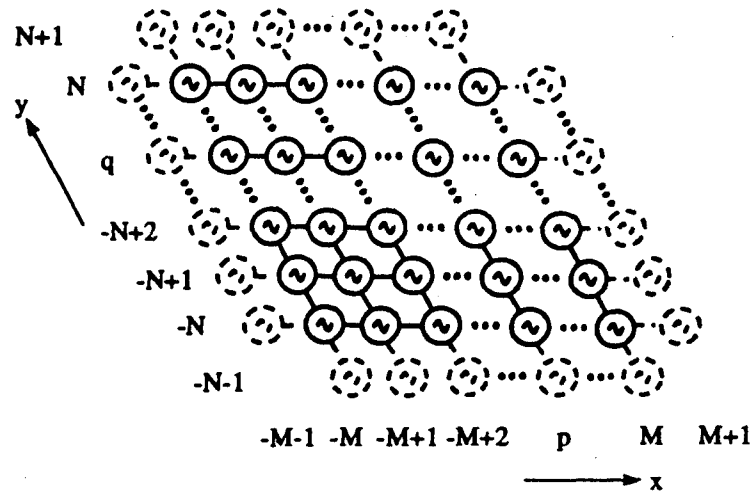
This diagram schematically represents a (2M+1) by (2N+1) array of oscillators coupled to nearest neighbors. This is the array to be analyzed in the following. The oscillators shown in dashed lines are fititious and their purpose will be described later in this presentation.

The Continuum Model

- Begin with Adler's theory applied to the array.

$$\frac{d\theta_{ij}}{dt} = \omega_{tune,ij} - \sum_{m=-M}^{M} \sum_{n=-N}^{N} \Delta\omega_{lock:ij,mn} \sin(\Phi_{ij,mn} + \theta_{ij} - \theta_{mn})$$

Define the phase by:

$$\theta_{ij} = \omega_{ref}t + \phi_{ij}$$

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To derive the continuum model of this two dimensional array, we begin with Adler's description of the injection locking phenomenon. In his theory, the time derivative of the phase of an injection locked oscillator is related to the sine of the phase difference between the oscillator signal and the injection signal. Generalizing this to the two dimensional array of mutually injection locked oscillators (with general interoscillator coupling topology) we arrive at the system of differential equations shown. We then define the phase, phi, as shown relative to a reference frequency which can be chosen arbitrarily.

The Continuum Model (Cont.)

Then,

$$\frac{d\phi_{ij}}{dt} = \omega_{nom,ij} - \omega_{ref} - \sum_{\substack{m=i-1\\m\ne i}}^{i+1} \sum_{\substack{n=j-1\\n\ne j}}^{j+1} \Delta\omega_{lock:ij,mn} \sin(\Phi_{ij,mn} + \phi_{ij} - \phi_{mn})$$

or,

$$\frac{d\phi_{ij}}{dt} = \omega_{nom,ij} - \omega_{ref} - \Delta\omega_{lock}\left[(\phi_{ij} - \phi_{i-1,j-1}) + (\phi_{ij} - \phi_{i-1,j+1}) + (\phi_{ij} - \phi_{i+1,j-1}) + (\phi_{ij} - \phi_{i+1,j+1})\right]$$

$$= \omega_{nom,ij} - \omega_{ref} + \Delta\omega_{lock}\left[\phi_{i-1,j-1} + \phi_{i-1,j+1} + \phi_{i+1,j-1} + \phi_{i+1,j+1} - 4\phi_{ij}\right]$$

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Using this definition of phi the system of equations become that shown here.
Then, assuming that the locking ranges are all the same, that the coupling
phase is zero, and that the phase differences between adjacent oscillators is
small, we can linearize the system as shown. Then, the quantity in the square
brackets can be identified as the finite difference approximation to the
Laplacian operator.

The Continuum Model (Cont.)

which leads to,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial \phi}{\partial \tau} = -\frac{\omega_{tune} - \omega_{ref}}{\Delta\omega_{lock}}$$

where,

$$\tau = \Delta\omega_{lock}\, t$$

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Thus, defining a continuous phi function and continuous variables x and y indexing the oscillators, we arrive at the partial differential equation for phi shown. This is the diffusion equation. Tau is time measured in inverse locking ranges.
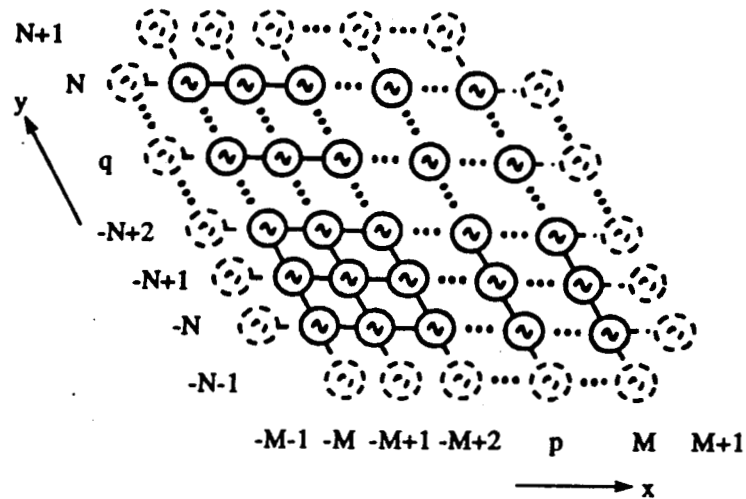
Boundary Conditions

- Employ an artifice proposed in the one dimensional case.
- Add fictitious oscillators on the periphery.
  - Dynamically tuned to reduce injection to zero.
  - Results in a Neumann condition on the boundary.

Having derived the differential equation governing the behavior of phi, we must determine the boundary conditions at the perimeter of the array in order to uniquely define the solution. For this we use an artifice in which fictitious oscillators are added on the periphery of the array and these are dynamically tuned in such a manner as to render the phase of each fictitious oscillator equal to its nearest real neighbor in the array. This effectively emulates the absence of the fictitious oscillator because when the phases are equal the injection effect on the dynamics is zero by Adler's theory. Now, the equality of the two phases implies a zero value for the derivative of phase normal to the array edge; i.e., a Neumann boundary condition.

The M by N Array

This diagram illustrates the fictitious oscillator arrangement used in the boundary condition derivation.

The Ensemble Frequency

Averaging over the array,

$$< \frac{\partial^2 \phi}{\partial x^2} > + < \frac{\partial^2 \phi}{\partial y^2} > - \frac{\partial <\phi>}{\partial \tau} = -\frac{< \omega_{tune} > - \omega_{ref}}{\Delta \omega_{lock}}$$

$$\frac{\omega}{\Delta \omega_{lock}} = \frac{d\phi}{d\tau} + \frac{\omega_{ref}}{\Delta \omega_{lock}}$$

$$< \omega > = < \omega_{tune} >$$

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The partial differential equation can now be used to determine the frequency at which the ensemble of mutually injection locked oscillators will oscillate without external injection. This is done by averaging the equation over the area of the array. The result indicates that the ensemble frequency will be equal to the average of the tuning (free running) frequencies.

The Green's Function

$$-\frac{\omega_{tune} - \omega_{ref}}{\Delta\omega_{lock}} = -Cu(\tau)\delta(x - x')\delta(y - y')$$

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} - sg = -\frac{C}{s}\delta(x - x')\delta(y - y')$$

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The equation will be solved by means of a Green's function, g; that is, a solution for the case of a delta function source term as shown. Performing a Laplace transformation with respect to the time variable results in the spatial equation shown.

The Eigenfunctions

$$f_{ee,mn} = \frac{1}{N_{ee,mn}} \cosh\left(\sqrt{s_m}\, x\right) \cosh\left(\sqrt{s_n}\, y\right)$$

$$f_{oo,k\ell} = \frac{1}{N_{oo,k\ell}} \sinh\left(\sqrt{s_k}\, x\right) \sinh\left(\sqrt{s_\ell}\, y\right)$$

$$f_{ee,m\ell} = \frac{1}{N_{eo,m\ell}} \cosh\left(\sqrt{s_m}\, x\right) \sinh\left(\sqrt{s_\ell}\, y\right)$$

$$f_{oo,kn} = \frac{1}{N_{oe,kn}} \sinh\left(\sqrt{s_k}\, x\right) \cosh\left(\sqrt{s_n}\, y\right)$$

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The Green's function can be expressed as a sum of eigenfunctions of the differential operator in the spatial equation. The eigenfunctions come in four types according to their even or odd symmetry with respect to the spatial variables.

The Eigenvalues

$$s_k = -\left(\frac{(2k+1)\pi}{2a+1}\right)^2 \qquad s_m = -\left(\frac{2m\pi}{2a+1}\right)^2$$

$$s_\ell = -\left(\frac{(2\ell+1)\pi}{2b+1}\right)^2 \qquad s_n = -\left(\frac{2n\pi}{2b+1}\right)^2$$

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Applying Neumann boundary conditions determines the eigenvalues.

Normalization

$$N_{ee,mn} = \frac{1}{2}\sqrt{(2a+1)(2b+1)\varepsilon_m \varepsilon_n}$$

$$N_{oo,k\ell} = \frac{1}{2}\sqrt{(2a+1)(2b+1)}$$

$$N_{eo,m\ell} = \frac{1}{2}\sqrt{(2a+1)(2b+1)\varepsilon_m}$$

$$N_{oe,kn} = \frac{1}{2}\sqrt{(2a+1)(2b+1)\varepsilon_n}$$

$$\varepsilon_m = \begin{array}{ll} 2; & m=0 \\ 1; & m\neq 0 \end{array}$$

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It will be convenient to normalize the eigenfunctions so that the integral of their absolute square over the area of the array is unity. The necessary normalization constants are shown here.

Normalized Eigenfunctions

$$\tilde{f}_{oo,mn} = \frac{1}{\sqrt{(2a+1)(2b+1)\varepsilon_m \varepsilon_n}} \cos\left(\frac{2m\pi x}{2a+1}\right)\cos\left(\frac{2n\pi y}{2b+1}\right)$$

$$\tilde{f}_{oo,kl} = \frac{1}{\sqrt{(2a+1)(2b+1)}} \sin\left(\frac{(2k+1)\pi x}{2a+1}\right)\sin\left(\frac{(2l+1)\pi y}{2b+1}\right)$$

$$\tilde{f}_{oo,ml} = \frac{1}{\sqrt{(2a+1)(2b+1)\varepsilon_m}} \cos\left(\frac{2m\pi x}{2a+1}\right)\sin\left(\frac{(2l+1)\pi y}{2b+1}\right)$$

$$\tilde{f}_{oo,kn} = \frac{1}{\sqrt{(2a+1)(2b+1)\varepsilon_n}} \sin\left(\frac{(2k+1)\pi x}{2a+1}\right)\cos\left(\frac{2m\pi y}{2b+1}\right)$$

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Using the preceding results, we obtain the above set of normalized eigenfunctions required for the expansion of the Green's function.

Back to the Green's Function

$$g(x, y; x', y'; s) = -\frac{C}{s}\left[G_{\alpha\epsilon} + G_{\sigma\sigma} + G_{\epsilon\sigma} + G_{\sigma\epsilon}\right]$$

$$G_{\alpha\epsilon}(x, y; x', y'; s) = \sum_{a=0}^{\infty}\sum_{m=0}^{\infty}\frac{4\cosh\left(x'\sqrt{s_m^*}\right)\cosh\left(y'\sqrt{s_a^*}\right)\cosh\left(x\sqrt{s_m}\right)\cosh\left(y\sqrt{s_a}\right)}{\varepsilon_a\varepsilon_m(2a+1)(2b+1)(s_m + s_a - s)}$$

$$G_{\sigma\sigma}(x, y; x', y'; s) = \sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\frac{4\sinh\left(x'\sqrt{s_k^*}\right)\sinh\left(y'\sqrt{s_l^*}\right)\sinh\left(x\sqrt{s_k}\right)\sinh\left(y\sqrt{s_l}\right)}{(2a+1)(2b+1)(s_k + s_l - s)}$$

$$G_{\epsilon\sigma}(x, y; x', y'; s) = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{4\cosh\left(x'\sqrt{s_m^*}\right)\sinh\left(y'\sqrt{s_l^*}\right)\cosh\left(x\sqrt{s_m}\right)\sinh\left(y\sqrt{s_l}\right)}{\varepsilon_m(2a+1)(2b+1)(s_m + s_l - s)}$$

$$G_{\sigma\epsilon}(x, y; x', y'; s) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{4\sinh\left(x'\sqrt{s_k^*}\right)\cosh\left(y'\sqrt{s_n^*}\right)\sinh\left(x\sqrt{s_k}\right)\cosh\left(y\sqrt{s_n}\right)}{\varepsilon_n(2a+1)(2b+1)(s_k + s_n - s)}$$

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Using Sturm-Liouville theory we can immediately write the Green's function in terms of the eigenfunctions. Each term of the expansion has one simple pole at the eigenvalue rendering the inverse Laplace transform a trivial matter of summing the residues at these poles. There is, however, a double pole at the origin which yields a term linear in time.

The Solution for Phase

$$\phi(x, y; x', y'; \tau) = \frac{C\tau}{(2a+1)(2b+1)} u(\tau)$$

$$+ \sum \sum \frac{4\cos\left(\frac{2\pi a x'}{2a+1}\right)\cos\left(\frac{2\pi b y'}{2b+1}\right)\cos\left(\frac{2\pi a x}{2a+1}\right)\cos\left(\frac{2\pi b y}{2b+1}\right)}{\varepsilon_a \varepsilon_b (2a+1)(2b+1)\left[\left(\frac{2\pi a \pi}{2a+1}\right)^2 + \left(\frac{2\pi \pi}{2b+1}\right)^2\right]}\left\{1 - e^{-\left[\left(\frac{2\pi a}{2a+1}\right)^2 + \left(\frac{2\pi \pi}{2b+1}\right)^2\right]}\right\} u(\tau)$$

$$+ \sum \sum \frac{4\sin\left(\frac{(2k+1)\pi x'}{2a+1}\right)\sin\left(\frac{(2l+1)\pi y'}{2b+1}\right)\sin\left(\frac{(2k+1)\pi x}{2a+1}\right)\sin\left(\frac{(2l+1)\pi y}{2b+1}\right)}{(2a+1)(2b+1)\left[\left(\frac{(2k+1)\pi}{2a+1}\right)^2 + \left(\frac{(2l+1)\pi}{2b+1}\right)^2\right]}\left\{1 - e^{-\left[\left(\frac{(2k+1)\pi}{2a+1}\right)^2 + \left(\frac{(2l+1)\pi}{2b+1}\right)^2\right]}\right\} u(\tau)$$

$$+ \sum \sum \frac{4\cos\left(\frac{2\pi a x'}{2a+1}\right)\sin\left(\frac{(2l+1)\pi y'}{2b+1}\right)\cos\left(\frac{2\pi a x}{2a+1}\right)\sin\left(\frac{(2l+1)\pi y}{2b+1}\right)}{\varepsilon_a (2a+1)(2b+1)\left[\left(\frac{2\pi a}{2a+1}\right)^2 + \left(\frac{(2l+1)\pi}{2b+1}\right)^2\right]}\left\{1 - e^{-\left[\left(\frac{2\pi a}{2a+1}\right)^2 + \left(\frac{(2l+1)\pi}{2b+1}\right)^2\right]}\right\} u(\tau)$$

$$+ \sum \sum \frac{4\sin\left(\frac{(2k+1)\pi x'}{2a+1}\right)\cos\left(\frac{2\pi b y'}{2b+1}\right)\sin\left(\frac{(2k+1)\pi x}{2a+1}\right)\cos\left(\frac{2\pi b y}{2b+1}\right)}{\varepsilon_b (2a+1)(2b+1)\left[\left(\frac{(2k+1)\pi}{2a+1}\right)^2 + \left(\frac{2\pi \pi}{2b+1}\right)^2\right]}\left\{1 - e^{-\left[\left(\frac{(2k+1)\pi}{2a+1}\right)^2 + \left(\frac{2\pi \pi}{2b+1}\right)^2\right]}\right\} u(\tau)$$

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This is the inverse Laplace transform giving the dynamic behavior of the phase function when one oscillator is step detuned by one locking range at time zero. A remaining issue is that, because the source is a delta function instead of a pulse over on unit cell of the array, this series diverges at the detuned oscillator.

Pulse Source Representation

$$\phi(x,y;x',y';\tau) = \frac{C\tau}{(2a+1)(2b+1)}u(\tau)$$

$$+\sum_{}\sum_{} \frac{4\cos\left(\frac{2\pi a x'}{2a+1}\right)\cos\left(\frac{2\pi b y'}{2b+1}\right)\cos\left(\frac{2\pi a x}{2a+1}\right)\cos\left(\frac{2\pi b y}{2b+1}\right)}{\varepsilon_n \varepsilon_s (2a+1)(2b+1)\left[\left(\frac{2\pi a}{2a+1}\right)^2 + \left(\frac{2\pi b}{2b+1}\right)^2\right]}\left\{1-e^{-\left\{\left(\frac{2\pi a}{2a+1}\right)^2 + \left(\frac{2\pi b}{2b+1}\right)^2\right\}}\right\}u(\tau)$$

$$\times \frac{\sin\left(\frac{a\pi\xi}{2a+1}\right)\sin\left(\frac{b\pi\xi}{2a+1}\right)}{\left(\frac{a\pi\xi}{2a+1}\right)\left(\frac{b\pi\xi}{2a+1}\right)}$$

+ three more double sums.

The divergence arising from the use of the Dirac delta representation can be circumvented by using a pulse source. The corresponding solution can be obtained by integrating the previous solution over one unit cell. The result is shown here and is tantamount to multiplication by appropriate sinc functions. This series converges everywhere in the array.

Oscillator Phases
Two Dimensional Array

Oscillator at (0,0) detuned one tuning range.
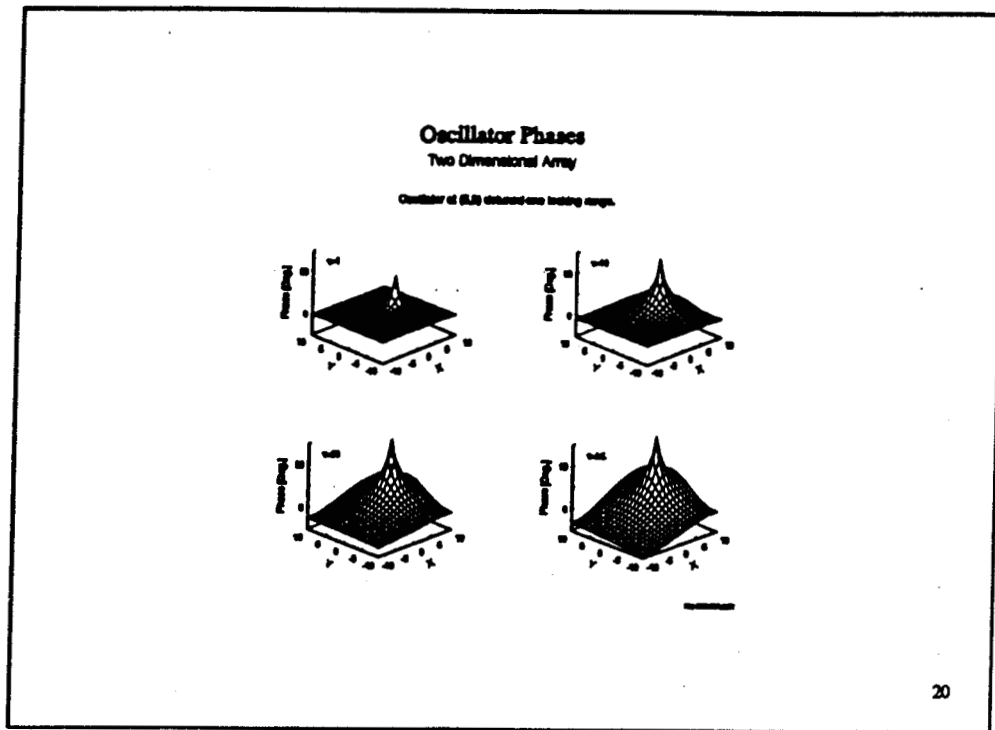
This is a graphical representation of the solution shown in the previous chart excluding the term linear in time. Four times are shown, the first near zero of time, the second part way into the transient, the third near convergence, and the fourth at infinite time. The excluded linear time term arises because the ensemble frequency changes in concert with the change in the average of the tuning frequencies of the oscillators resulting from the detuning of one of them.

The Array Time Constants

$$\tau_0 = \frac{1}{\pi^2}\left[\frac{(2a+1)^2(2b+1)^2}{(2a+1)^2+(2b+1)^2}\right]$$

General

$$\tau_0 = \frac{1}{\pi^2}\left[\frac{(2a+1)^2(2b+1)^2}{(2a+1)^2+4(2b+1)^2}\right]$$

On y axis

$$\tau_0 = \frac{1}{\pi^2}\left[\frac{(2a+1)^2(2b+1)^2}{4(2a+1)^2+(2b+1)^2}\right]$$

On x axis

$$\tau_0 = \frac{1}{4\pi^2}\left[\frac{(2a+1)^2(2b+1)^2}{(2a+1)^2+(2b+1)^2}\right]$$

Center

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The solution permits recognition of the slowest time constant which is representative of the response time (bandwidth) of the whole array. Four cases are possible resulting in slightly different results. However, in general these time constants are roughly proportional to the number of elements in the array.

Beamsteering

$$\frac{\omega_{tune}}{\Delta\omega_{lock}} = \frac{\omega_{ref}}{\Delta\omega_{lock}} + \left[\Omega_{x_1}\delta(x'-c_1) + \Omega_{x_2}\delta(x'-c_2)\right.$$

$$\left. + \Omega_{y_1}\delta(y'-d_1) + \Omega_{y_2}\delta(y'-d_2)\right]u(\tau)$$

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Beamsteering requires planar phase distributions over the array area. This can be obtained by detuning the perimeter elements according to the prescription shown here. Note that only four voltages are required since all of the oscillators along a given array edge are detuning by equal amounts.

The Dynamic Solution

$$\phi(x,y,\tau) = \left(\frac{\Omega_1+\Omega_2}{2a+1}+\frac{\Omega_3+\Omega_4}{2b+1}\right)\tau\, u(\tau)$$

$$+\frac{\Omega_1}{(2a+1)}\sum_{i=1}^{\infty}\frac{\cos\left[\frac{p\pi}{2a+1}(x-c_i)\right]+(-1)^p\cos\left[\frac{p\pi}{2a+1}(x+c_i)\right]}{\left(\frac{p\pi}{2a+1}\right)^2}\left[1-e^{-\left(\frac{p\pi}{2a+1}\right)^2\tau}\right]u(\tau)$$

$$+\frac{\Omega_2}{(2a+1)}\sum_{i=1}^{\infty}\frac{\cos\left[\frac{p\pi}{2a+1}(x-c_i)\right]+(-1)^p\cos\left[\frac{p\pi}{2a+1}(x+c_i)\right]}{\left(\frac{p\pi}{2a+1}\right)^2}\left[1-e^{-\left(\frac{p\pi}{2a+1}\right)^2\tau}\right]u(\tau)$$

$$+\frac{\Omega_3}{(2b+1)}\sum_{i=1}^{\infty}\frac{\cos\left[\frac{p\pi}{2b+1}(y-d_i)\right]+(-1)^p\cos\left[\frac{p\pi}{2b+1}(y+d_i)\right]}{\left(\frac{p\pi}{2b+1}\right)^2}\left[1-e^{-\left(\frac{p\pi}{2b+1}\right)^2\tau}\right]u(\tau)$$

$$+\frac{\Omega_4}{(2b+1)}\sum_{i=1}^{\infty}\frac{\cos\left[\frac{p\pi}{2b+1}(y-d_i)\right]+(-1)^p\cos\left[\frac{p\pi}{2a+1}(y+d_i)\right]}{\left(\frac{p\pi}{2b+1}\right)^2}\left[1-e^{-\left(\frac{p\pi}{2b+1}\right)^2\tau}\right]u(\tau)$$

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The solution resulting from the beamsteering detuning prescription can be obtain by integrating the source function multiplied by the Green's function obtained previously. The result is shown here.

The Steady State Solution

$$\phi(x, y; \tau) = \left(\frac{\Omega_{x_2} + \Omega_{x_1}}{2a+1} + \frac{\Omega_{y_2} + \Omega_{y_1}}{2b+1} \right) \tau$$

$$+ \left[\frac{\Omega_{x_2} + \Omega_{x_1}}{2} \right] \left[\frac{2a+1}{6} + \frac{1}{2}\left(\frac{c_2^2 + c_1^2}{2a+1} \right) - \frac{1}{2}\left(|x - c_2| + |x - c_1| \right) + \frac{x^2}{2a+1} \right]$$

$$+ \left[\frac{\Omega_{x_2} - \Omega_{x_1}}{2} \right] \left[\frac{1}{2}\left(\frac{c_2^2 - c_1^2}{2a+1} \right) + \frac{1}{2}\left(|x - c_2| - |x - c_1| \right) \right]$$

$$+ \left[\frac{\Omega_{y_2} + \Omega_{y_1}}{2} \right] \left[\frac{2b+1}{6} + \frac{1}{2}\left(\frac{d_2^2 + d_1^2}{2b+1} \right) - \frac{1}{2}\left(|y - d_2| + |y - d_1| \right) + \frac{y^2}{2b+1} \right]$$

$$+ \left[\frac{\Omega_{y_2} - \Omega_{y_1}}{2} \right] \left[\frac{1}{2}\left(\frac{d_2^2 - d_1^2}{2b+1} \right) + \frac{1}{2}\left(|y - d_2| - |y - d_1| \right) \right]$$

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The steady state solution can be derived from the dynamic solution by noting that, at infinite time, the summations can be performed in closed form resulting in quadratic functions. The form above clearly shows that the symmetric part of the detuning gives quadratic phase dependence while the antisymmetric part yields the linear dependence desired for beamsteering.

Parameters for Beamsteering

$$c_1 = -c_2 = -c$$

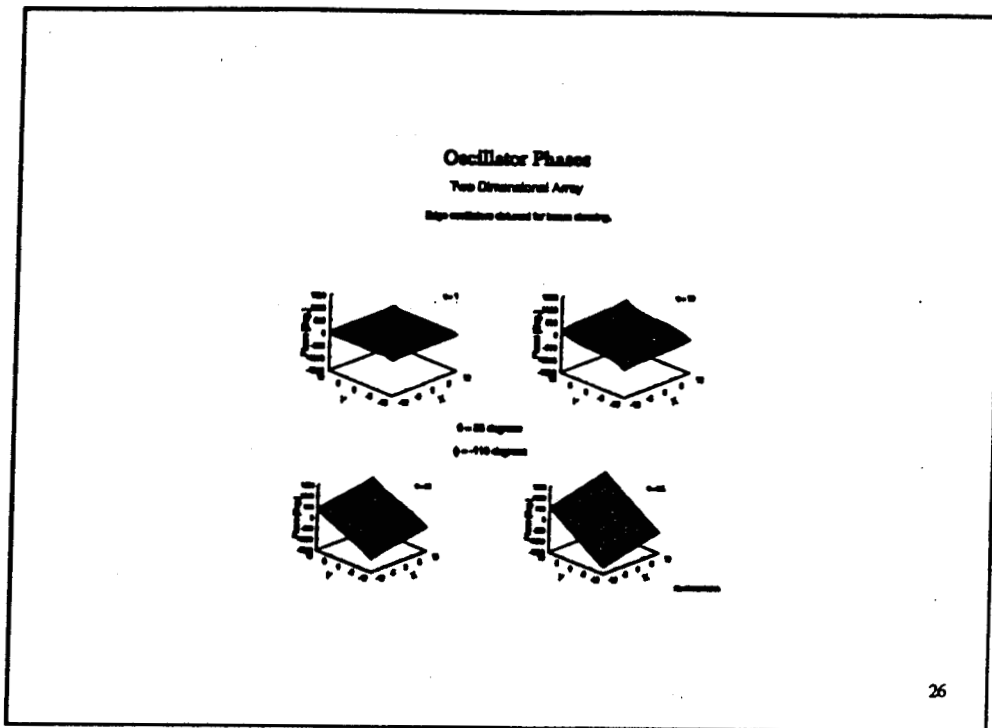$$d_1 = -d_2 = -d$$

$$\Omega_{x_1} = -\Omega_{x_2} = -\Omega_x$$

$$\Omega_{x_1} = -\Omega_{x_2} = -\Omega_x$$

which yields,

$$\phi(x, y; \tau) = \frac{\Omega_x}{2}(|x+d|-|x-d|) + \frac{\Omega_y}{2}(|y+d|-|y-d|)$$

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By selecting the parameters for antisymmetric detuning as shown, we obtain the desired linear phase.

These graphs show the time evolution of the phase when detuning appropriate to beamsteering is applied.
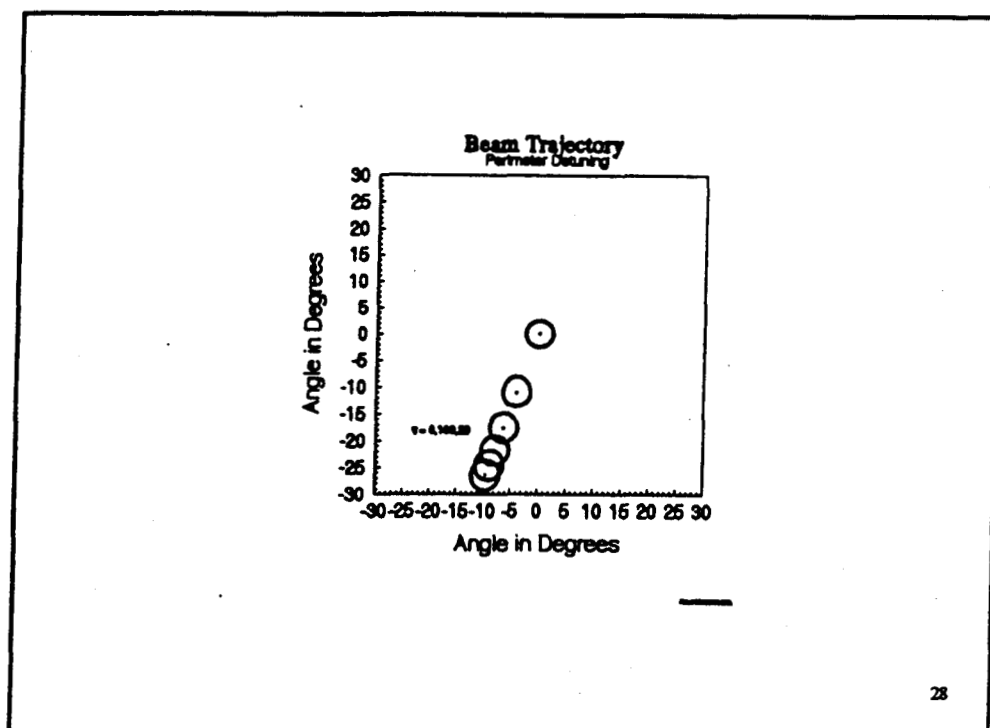
Radiating Aperture

$$\Omega_x = 2\pi \frac{h}{\lambda} \sin\theta_0 \cos\phi_0$$

$$\Omega_y = 2\pi \frac{h}{\lambda} \sin\theta_0 \sin\phi_0$$

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If we consider a radiating aperture composed of elements spaced by distance h in a two dimensional square lattice, the tuning necessary to steer the beam to desired polar angles is given by the above formulae.

Beam Trajectory
Perimeter Detuning

This graph shows the beam peak (dots) and the three dB contour (closed curves) as a function of time during the beamsteering transient resulting when a step steering voltage designed to steer the beam thirty degrees off normal is applied at time zero.

Peak Gain Dynamics
Perimeter Detuning
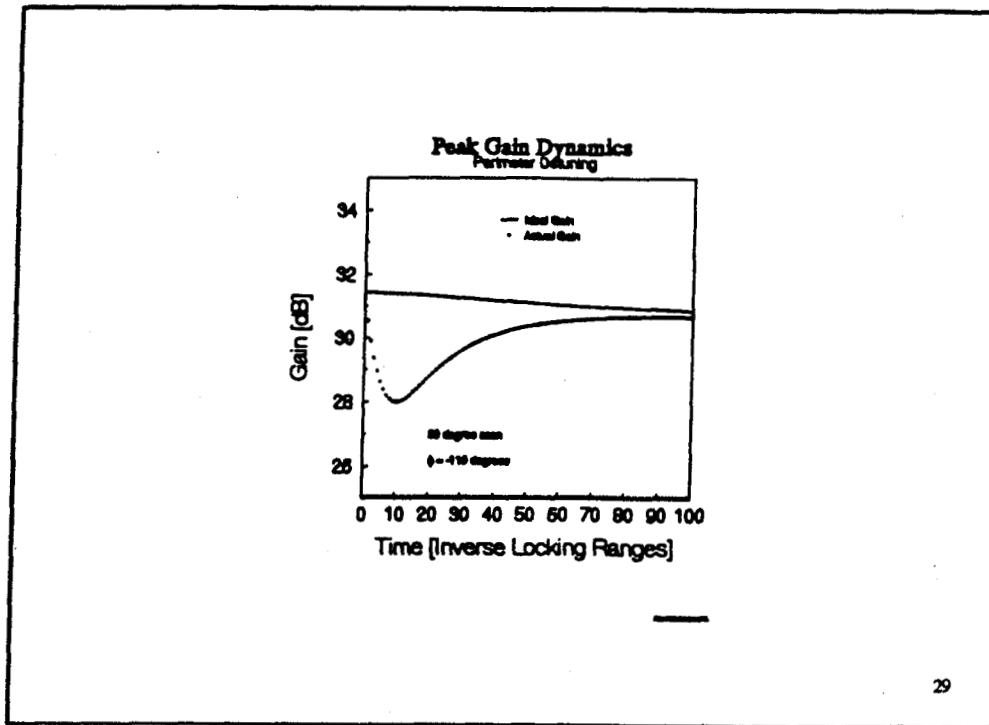
During the transient period, the aperture phase is nonplanar. This results in a temporary reduction in gain due to phase aberration. This graph shows this gain reduction as a function of time compared with the projected aperture loss to be expected for each beam position. These curves were obtained by pattern integration.

Beam Trajectory
Perimeter Detuning
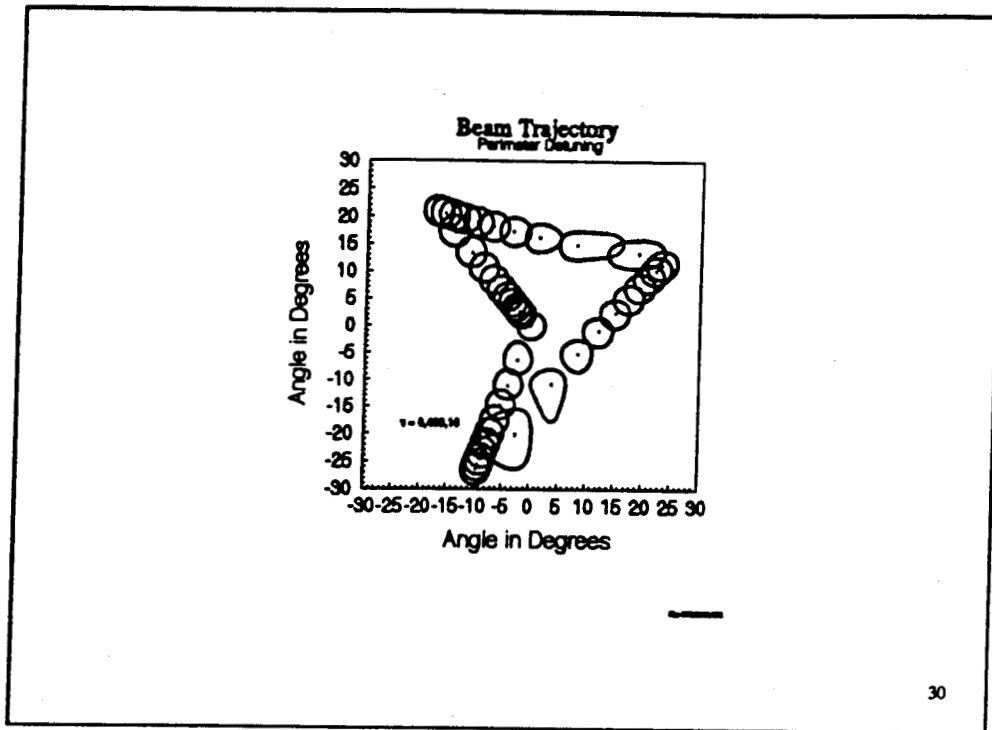
This graph shows the result of four sets of steering voltages applied in rapid succession. Note that the aberration effects seem to be greater when steering from one off axis position to another than when steering to or from normal.

Concluding Remarks

- Inter-oscillator phase difference
  - Limited to 90 degrees.
  - Limit can be mitigated by:
    - Reducing the element spacing.
    - Adding oscillators between the radiating ones.
    - Radiating at a harmonic of the coupling frequency.
- This techniques appears to hold promise for simplification of the beamsteering control system.

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One limitation of the present system is that the phase difference between adjacent oscillators is limited to 90 degrees to maintain lock. (The validity of the linearized theory actually requires that the phase difference be small compared to 90 degrees.) This would appear to limit the scan of a radiating aperture with half wavelength element spacing to 30 degrees off axis. However, this can be mitigated in several ways. One can reduce the spacing between the elements, one can radiate only from every second or every third oscillator, or one can radiate at a harmonic of the coupling frequency.

Overall , this appear to be an interesting technique for beamsteering which results in considerable simplification of the steering electronics in that it only requires four analog voltages to achieve steering in two dimensions.